DEFENCE SERVICES ACADEMY
ENTRANCE EXAMINATION
MATHEMATICS

Date: 18-8-2019 **Time Allowed: 2 Hours**

ANSWER ALL QUESTIONS
PART (A)

1. Choose the correct or the most appropriate answer for each question.
Write the letter of the correct or the most appropriate answer. **(22 Marks)**

(1) Functions f and g are given by $f(3)=-1$ and $g(-1)=5$. Then $(g\circ f)^{-1}(5)=$

A. -1 B. 3 C. 4 D. 5 E. 0

(2) When $(2x+k)^{2019}+(x-1)^2$ is divided by $x+1$, the remainder is 5, then $k=$

A. -1 B. 1 C. -3 D. 6 E. 3

(3) If ${}^nC_2=66$, then $n=$

A. 9 B. 10 C. 11 D. 12 E. 13

(4) The product of the A.M. and G.M. between 4 and 16 is

A. 40 B. 60 C. 70 D. 80 E. 160

(5) Let $A=\begin{pmatrix}1 & 2\\ 0 & 4\end{pmatrix}$ be a matrix and given that $\det(xA)=4$. Then $x=$

A. 0 B. ±1 C. ±2 D. 3 E. 4

(6) If A is an event such that $6[P(A)]^2=P(\text{not } A)$, then $P(A)=$

A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{1}{6}$ D. $\frac{2}{3}$ E. none of these

(7) Chords AB and CD of a circle intersect at P within the circle. If $AP=5$, $PB=2, CP=x$ and $PD=x+3$, then $x=$

A. 2 B. 3 C. 4 D. 5 E. 6

(8) The areas of two similar triangles are in the ratio $4:9$. One side of the smaller triangle is 4. The corresponding side of the other triangle is

A. 2 B. 3 C. 4 D. 5 E. 6

(9) If $\vec{a},\vec{b}$ are non-parallel and non-zero such that $(3x+y)\vec{a}+(y-3)\vec{b}=\vec{0}$, then $x=$

A. 1 B. -1 C. 3 D. -3 E. none of these

(10) What is the smallest value of x for which $\tan 3x=-1$?

A. 15° B. 45° C. 75° D. 90° E. 105°

(11) If $f(x)=4x^2+e^{-3x}$, then $f''(0)=$

A. -17 B. 8 C. 17 D. -8 E. -3

PART (B)

2. (a) Functions $f : R \rightarrow R$ and $g : R \rightarrow R$ are defined by $f(x) = 2x - 1$ and $g(x) = 4x + 3$. Find the value of x for which $(f^{-1} \circ g)(x) = (g^{-1} \circ f)(x) + 6$.

(6 marks)

(b) The expression $x^3 + ax^2 + bx + 3$ is exactly divisible by $x + 3$ but it leaves a remainder of 91 when divided by $x - 4$. What is the remainder when it is divided by $x + 2$? **(7 marks)**

3. (a) In the expansion of $(1 - 2x)^n$, the sum of the coefficients of x and x^2 is 16. Given that n is positive, find the value of n and the coefficient of x^3.

(6 marks)

(b) Use a graphical method to find the solution set of the inequation $2x(x - 1) < 3 - x$ and illustrate it on the number line. **(7 marks)**

4. (a) The product of first three terms of a G.P. is 1000. If we add 6 to its second term , 7 to its third term and its first term is not changed, then three terms form an A.P.. Find the first three terms of the G.P. . **(6 marks)**

(b) Find the inverse of the matrix $A = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$ and investigate whether or not the squares of A and A^{-1} are inverses of each other. **(7 marks)**

5. (a) How many 3-digit numerals can you form from $3, 0, 1$ and 6 without repeating any digit? Find the probability of an even number and find the probability that a numeral which is divisible by 3. **(6 marks)**

(b) PQR is a triangle in which $PQ = PR$. S is a point inside the triangle such that $\angle SPQ = \angle SQR$. T is the point on QS such that $PT = PS$. Prove that $PQRT$ is cyclic. **(7 marks)**

6. (a) P, Q, R, S are four points in order on a circle O, so that PQ is a diameter. PS and QR meet at T. If $\alpha(PQRS) = 3\alpha(\Delta TRS)$, prove that $\angle ROS = 60^{\circ}$.

(6 marks)

(b) Find the matrix which will translate through 3 units horizontally and 1 unit vertically followed by a rotation through 45°, and find the map of the point $(1, 2)$. **(7 marks)**

7. (a) If $x + y + z = \pi$, prove that $\sin 2x + \sin 2y + \sin 2z = 4 \sin x \sin y \sin z$.

(6 marks)

(b) If the perimeter of a rectangle is $24\,m$, show that the area is the greatest when this rectangle is a square and find the maximum area. **(7 marks)**