DEFENCE SERVICES ACADEMY
ENTRANCE EXAMINATION
MATHEMATICS

Date: 19-8-2018 **Time Allowed: 2 Hours**

ANSWER ALL QUESTIONS
PART (A)

1. Choose the correct or the most appropriate answer for each question.
Write the letter of the correct or the most appropriate answer. **(22 Marks)**

(1) Functions f and g are given by $f(x)=2x$ and $g(x)=x+3$. If $(g\circ f)^{-1}(t)=1$, then $t=$

A. -5 B. -3 C. 2 D. 3 E. 5

(2) It is given that the remainder is 178 when x^n-5x^2-20 is divided by $x-3$, then the value of n is

A. -4 B. 4 C. 3 D. -3 E. 5

(3) ${}^nC_0+{}^nC_1+{}^nC_n=$

A. n B. $n+1$ C. 2 D. $n+2$ E. none of these

(4) Given that $7,a,b,c,-5$ in an A.P., then the mean of a,b,c is

A. -2 B. 1 C. $\frac{3}{2}$ D. 3 E. 4

(5) The matrix $M=\begin{pmatrix} a & 4 \\ 16 & b \end{pmatrix}$ is singular and a,b are positive integers. Then $a+b$ cannot be

A. 16 B. 20 C. 34 D. 48 E. 65

(6) If A is an event such that $P(A)=x$ and $P(\text{not}\,A)=y$, then $x^3+y^3=$

A. $3xy$ B. $1+3xy$ C. $3xy-1$ D. $1-3xy$ E. none of these

(7) Chords AB and CD of a circle intersect at P within the circle. If $AP=x$, $PB=x-2$, $CP=8$ and $PD=3$, then $x=$

A. 2 B. 3 C. 4 D. 5 E. 6

(8) If $\Delta ABC \sim \Delta PQR$, $\alpha(\Delta ABC)+\alpha(\Delta PQR)=75\text{cm}^2$, AB and PQ are corresponding sides and $AB:PQ=4:3$, then $\alpha(\Delta ABC)$, in cm^2, is

A. 25 B. 27 C. 36 D. 48 E. 50

(9) Given that $\vec{a}=3\hat{i}+4\hat{j}$. Then the vector with magnitude 20 units and in the direction of $\vec{a}$ is

A. $9\hat{i}+12\hat{j}$ B. $60\hat{i}+120\hat{j}$ C. $21\hat{i}+28\hat{j}$ D. $12\hat{i}+16\hat{j}$ E. $-12\hat{i}-16\hat{j}$

(10) If A,B,C are the angles of a triangle and $\tan A=3$ and $\tan B=2$, then $\tan C=$

A. 1 B. 2 C. 3 D. 4 E. 5

(11) The gradient of the tangent line to the curve $y=ax^2-4x+3$ at the point $x=1$ is -2. The value of a is

A. 3 B. 2 C. 1 D. -3 E. 4

P.T.O. →

PART (B)

2. (a) The functions f and g are defined for real x by $f(x)=2x-1$ and $g(x)=2x+3$. Evaluate $(g^{-1}\circ f^{-1})(2)$. **(6 marks)**

(b) Given $f(x)=x^3+px^2-2x+4\sqrt{3}$ has a factor $x+\sqrt{2}$, find the value of p. Show that $x-2\sqrt{3}$ is also a factor and solve the equation $f(x)=0$.

(7 marks)

3. (a) If the 2^{nd} and the 3^{rd} term in $(a+b)^n$ are in the same ratio as the 3^{rd} and 4^{th} in $(a+b)^{n+3}$, then find n. **(6 marks)**

(b) Use graphical method to find the solution set of the inequation $2x(x-1)<3-x$ and illustrate it on the number line. **(7 marks)**

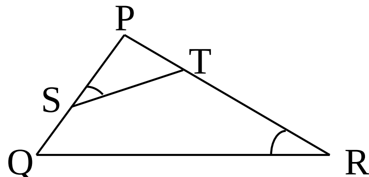
4. (a) The three numbers a,b,c between 2 and 18 are such that their sum is 25, the numbers $2,a,b$ are consecutive terms of an arithmetic progression, and the numbers $b,c,18$ are consecutive terms of a geometric progression. Find the three numbers. **(6 marks)**

(b) Find the inverse of $\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$ by using the definition of inverse of matrix. **(7 marks)**

5. (a) A die is rolled 360 times. Find the expected frequency of a factor of 6 and the expected frequency of a prime number. If all the scores obtained in these 360 trails are added together, what is the expected total score? **(6 marks)**

(b) PQR is a triangle in which $PQ=PR$. S is a point inside the triangle such that $\angle SPQ=\angle SQR$. T is the point on QS produced such that $PT=PS$. Prove that $PQRT$ is cyclic. **(7 marks)**

6. (a) In the figure $\angle PST=\angle PRQ$, $PS:SQ=3:1$ and $PT:TR=1:2$. If $PT=2$, find the length of PS and the ratios of $\alpha(\Delta PST):\alpha(\Delta PQR)$ and $\alpha(\Delta PST):\alpha(QRTS)$. **(6 marks)**



(b) The position vectors of A and B relative to an origin O are $\begin{pmatrix}5\\15\end{pmatrix}$ and $\begin{pmatrix}13\\3\end{pmatrix}$ respectively. Given that C lies on AB and has position vector $\begin{pmatrix}2t+1\\t+1\end{pmatrix}$, find the value of t and the ratio $AC:CB$. **(7 marks)**

7. (a) If $x+y+z=\pi$, show that

$$\cos\frac{x}{2}+\cos\frac{y}{2}+\cos\frac{z}{2}=4\cos\frac{y+z}{4}\cos\frac{z+x}{4}\cos\frac{x+y}{4}.$$ **(6 marks)**

(b) If $y=\ln(\cos 2x)$, prove that $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+4=0$. **(7 marks)**