DEFENCE SERVICES TECHNOLOGICAL ACADEMY
ENTRANCE EXAMINATION
MATHEMATICS

Date: 18-8-2018 **Time Allowed: 2 Hours**

Attempt All Questions

1. Choose the correct or the most appropriate answers for each question. Write only the letter of the answer. **(30 Marks)**

(1) $f:R\mapsto R,\ g:R\mapsto R,\ f(x)=3+2x^2, g(x)=2x+1,$ g maps f(x) onto 35, then $x=$

A. 7 B. 3 C. $\pm\sqrt{7}$ D. $\pm$ 3 E. $\pm\sqrt{3}$

(2) $f:R\mapsto R,\ f(x)=2^x$. If (f.f) (a) = 256, then a =

A. 1 B. 2 C. 3 D. 0 E. -1

(3) A function f is defined by $f:x\mapsto\frac{2x+2}{x-1}, x\neq 1$. Then $f^{-1}(3)$ is

A. 2 B. 3 C. 4 D. 5 E. 6

(4) Given that $(3x+k)^3+(4x-7)^2$ has a remainder 33 when divided by $x-3$ then $k=$

A. 2 B.-2 C. 7 D. -7 E. -11

(5) When $(x+k)^{2006}+(2x+1)^2$ is divided by x + 2 remainder is 10, then k =

A. 1 only B. -1 only C. 3 only D. 3 or 1 E. -1 or -3

(6) The expression $x^3+x^2-11x+4$ has a factor

A. $x-4$ B. $x-2$ C. $x+2$ D. $x+1$ E. $x+4$

(7) In the expansion of $(2x-\frac{1}{2}y)^6$, the coefficient of x^3y^3 is

A. -20 B. -80 C. 20 D. 80 E. -40

(8) In the expansion of $(1+kx)^{20}$, the coefficient of x^2 is 19. The positive value of k is

A. $\frac{1}{10}$ B. $\frac{1}{\sqrt{10}}$ C. 10 D. $\sqrt{10}$ E. $-\frac{1}{\sqrt{10}}$

(9) The solution set in R for the inequation $x^2-4x+4<0$ is

A. $\{x/x<2\}$ B. $\{x/x>2\}$ C. $\{2\}$ D. R E. ϕ

(10) The parabola $y = 6x^2 - 13x + 6$ cuts the X – axis at A and B. Then AB =

A. 0 B. $-\frac{5}{6}$ C. $\frac{5}{6}$ D. $\frac{13}{6}$ E. $-\frac{13}{6}$

(11) $1+\cos 45^\circ+\cos^2 45^\circ+\cos^3 45^\circ+\ldots.=$

A. 1 B. $\sqrt{2}-2$ C. $2-\sqrt{2}$ D. $1+\sqrt{2}$ E. $2+\sqrt{2}$

(12) For two numbers a and b, the A.M is 3 and G.M is $2\sqrt{2}$. Then $a^2+b^2=$

A. 36 B. 16 C. 20 D. 6 E. 24

(13) In a certain sequence if $u_1=1, u_2=2$ and $u_{n+2}=u_n\,u_{n+1}$, then $u_6=$

A. 12 B. 8 C. 32 D. 6 E. 16

(14) $\det\begin{bmatrix}a & b\\ c & d\end{bmatrix}=4$ and abcd = 192. Then $\det\begin{bmatrix}a^2 & b^2\\ c^2 & d^2\end{bmatrix}=$

A. 28 B. -28 C. $\pm$ 112 D. 256 E. 144

(15) If A is a 2 x 2 matrix such that det A = k and p is a real number then det (pA) =

A. pk B. p^2k C. pk^2 D. p^2k^2 E. p

(16) Let A be an event such that P(A will occur) = p and P(A will not occur) = q and $0<p<1$. If the sum to infinity of the G.P $p+p^2+p^3+---$ is x, then the sum to infinity of the G.P $q+q^2+q^3+---$ is

A. $1-x$ B. $1+x$ C. $\frac{1}{x}$ D. $\frac{1}{1-x}$ E. $\frac{1}{1+x}$

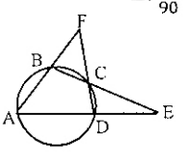
(17) Let P = {1, 2} and Q = {a, b}. If a function from P to Q is defined at random, the probability that it will be a one-to-one correspondence is

A. 1 B. $\frac{3}{4}$ C. $\frac{1}{4}$ D. $\frac{1}{2}$ E. None of these

(18) The entry of a matrix of order 4 x 5 is chosen at random. The probability that it will be from the second row or the third column is

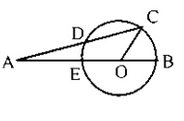
A. $\frac{1}{20}$ B. $\frac{1}{5}$ C. $\frac{1}{4}$ D. $\frac{2}{5}$ E. $\frac{2}{90}$

(19) In the figure $\angle A=56^\circ, \angle F=30^\circ$, Then $\angle E=$



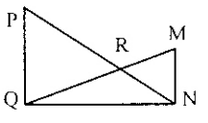
A. 56° B. 30° C. 86° D. 38° E. 48°

(20) In the figure $AE=2$, $AD=3$, $DC=5$. Then $OC=$



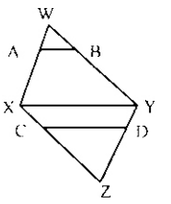
A. 4 B. 5 C. 6 D. 7 E. 8

(21) In the figure if $RM=6cm$, $PQ//MN$ and $\alpha(\Delta PQR):\alpha(\Delta RMN)=9:4$ then QR=



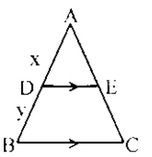
A. 4cm B. 9cm C. 12cm D. 15cm E. 5cm

(22) $CD//XY, AB//XY$, W and Z are equidistant from XY. If $AB = a$, $CD = 3a$ then $\frac{\alpha(\Delta AWB)}{\alpha(\Delta CZD)} =$



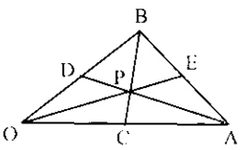
A. a^2 B. $\frac{1}{9}a^2$ C. $\frac{1}{9}$ D. $\frac{1}{3}$ E. $\frac{1}{3}a$

(23) In the figure if $\alpha(\Delta ADE) : \alpha(BCED) = 4:21$, then $x : y =$



A. 5:2 B. $2:\sqrt{21}$ C. 2:5 D. 4:25 E. 2:3

(24) In the figure C, D, E are the mid-points of OA, OB, AB respectively, If $\overrightarrow{OA} = 2\vec{a}$, $\overrightarrow{OB} = 2\vec{b}$, then $\overrightarrow{PC} =$



A. $\frac{1}{3}(2\vec{a}-\vec{b})$ B. $\frac{1}{3}(2\vec{a}+\vec{b})$ C. $\frac{1}{3}(\vec{b}-2\vec{a})$ D. $\frac{1}{3}(\vec{a}-2\vec{b})$ E. $\frac{1}{3}(\vec{a}+2\vec{b})$

(25) $\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + ... + \cos 180^\circ =$

A. 0 B. 1 C. -1 D. 2 E. -2

(26) $\frac{\tan 80^\circ - \tan 20^\circ}{1 + \tan 80^\circ \tan 20^\circ} =$

A. $\frac{1}{\sqrt{3}}$ B. $\frac{2}{\sqrt{3}}$ C. $\frac{1}{2}$ D. $\sqrt{3}$ E. $-\sqrt{3}$

(27) If $0^\circ \le \theta \le 360^\circ$, the number of elements in the solution set of $\sin\theta = 0$ is

A. 0 B. 1 C. 2 D. 3 E. 4

(28) The distance s cm of a particle at the end of t sec is given by $s = 12 + 30t - 2t^2$. The speed of the particle at the end of 2 sec is

A. 64 cm/sec B. 38 cm/sec C. 22 cm/sec D. 30 cm/sec E. 8 cm/sec

(29) $\lim_{x\to\infty} \frac{\sqrt{x^2+1}-x}{x} =$

A. 2 B. 1 C. 0 D. -1 E. -2

(30) $\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta} =$

A. 0 B. 1 C. 2 D. -1 E. -2

2. Three functions are given as follows; $f: x \to \frac{5x+k}{x-3}, x \neq 3, g: x \to x - 1, h: x \to ax^2 + b$

(i) Find the value of k such that there is only one value of x that satisfies the equation $f(x) = x$.

(ii) If $(h.g): x \mapsto 2x^2 - 4x - 1$, find the numerical value of a and b.

(iii) Using the value of k in part (i), express $g.f^{-1}$ in similar form. **(10 Marks)**

3. Given that $f(x) = 2x^3 + 2x^2 - 3x - 5$

(i) Evaluate f (1).

(ii) Find the remainder when f (x-3) is divided by (x-4),

(iii) Deduce the remainder when f (x^2-6) is divided by (x^2-8). **(10 Marks)**

4. The second, sixth and eighth terms of an A.P are three distinct consecutive terms of a G.P. Find the common ratio of the G.P. If the sum of the first five terms of G.P is 31, find its first term. **(10 Marks)**

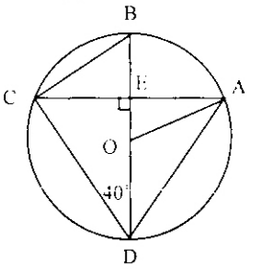
5. If a die rolled 60 times, what is the expected frequency of

(i) 4 turns up,

(ii) prime number turns up

(iii) a factor of 6 turns up. **(10 Marks)**

6. O is the centre and DB is the diameter of the circle. The line AC is perpendicular to BD and $\angle BDC = 40^\circ$. Find (a) $\angle DCA$, (b) $\angle ADB$, (c) $\angle CAO$. **(10 Marks)**



7. If $\sin 2\theta \neq 0$, prove that, $\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = 2$. Hence or otherwise, prove that $\sin^2 3\theta \operatorname{cosec}^2\theta - \cos^2 3\theta \sec^2\theta = 8\cos 2\theta$ **(10 Marks)**

8. Given the curve $ax^2 + by^2 = a + b$ where a and b are positive constants, find the coordinates of the points on the curve at which $\frac{dy}{dx} = 1$. **(10 Marks)**

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