DEFENCE SERVICES TECHNOLOGICAL ACADEMY
ENTRANCE EXAMINATION
MATHEMATICS

Date 17-8-2019 **Time Allowed: 2 Hours**

Attempt All Questions

1. Choose the correct or the most appropriate answers for each question. Write only the letter of the answer. **(30 Marks)**

(1) If $f(x+1) = x^2 + 2x + 4$, then $f(x) =$
A. $x^2 + 3$ **B.** $x^2 + 1$ **C.** $x^2 + 2x + 1$ **D.** $x^2 - 1$ **E.** $x^2 - 3$

(2) Given that $f(x) = \frac{13}{2-5x}$, $x \neq \frac{2}{5}$, then $f^{-1}(-1) =$
A. 1 **B.** 2 **C.** 3 **D.** 4 **E.** 5

(3) The function $f: R \to R$ is such that $f(x) = 2^x$. If $f(2) = (\sqrt{2})^p$, then $p =$
A. 1 **B.** 2 **C.** 3 **D.** 4 **E.** -2

(4) If $x + 2$ is a factor of $x^3 - ax^2 - 4x + 4$, then $a^2 - 1 =$
A. 0 **B.** 8 **C.** 15 **D.** 10 **E.** 3

(5) If *n* is an integer, then the remainder when $3x^{2n+3} - 4x^{2n+2} + 5x^{2n+1} - 8$ is divided by *(x+1)* is
A. -4 **B.** -10 **C.** 0 **D.** -20 **E.** 20

(6) In the Binomial expansion of $(1 - \frac{1}{10})^n$, the sum of 2nd and 3rd terms is Zero. Then $n =$
A. 20 **B.** 21 **C.** 22 **D.** 23 **E.** non of these

(7) The coefficient of x^2y^2 in the expansion of $(2x-3y)^4$ is
A. 72 **B.** 36 **C.** -36 **D.** 216 **E.** non of these

(8) The term independent of x in the expansion of $(\frac{1}{2x^2} - x)^9$ is
A. $\frac{21}{2}$ **B.** $\frac{23}{2}$ **C.** $\frac{30}{7}$ **D.** $\frac{25}{3}$ **E.** $\frac{18}{7}$

(9) The parabola $y = b + ax - x^2$ cuts the *x* axis at (-2,0) and (5,0). Then the solution set of $b + ax - x^2 < 0$ is
A. $\{x \backslash x < -2 \text{ or } x > 5\}$ **B.** $\{x \backslash x < 5 \text{ or } x > -2\}$ **C.** $\{x - 2 < x < 5\}$ **D.** $\{x - 5 \leq x \leq 2\}$ **E.** non of these

(10) The solution set in *R* for the inequation $x^2 - 4x + 4 \leq 0$ is
A. $\{2\}$ **B.** R **C.** $\emptyset$ **D.** $\{x - 2 \leq x \leq 2\}$ **E.** $\{x \backslash x \leq -2 \text{ or } x \geq 2\}$

(11) Let S_5 be the sum to the first 5 terms of an A.P and S^* be the sum of the next 5 terms. If $S^*-S_5=75$, then the common difference of that A.P is
A. 1 **B.** 2 **C.** 3 **D.** 4 **E.** 5

(12) If the A.M between two positive numbers *x* and *y* is 8 and the G.M between these numbers is 4 then $x^2+y^2=$
A. 224 **B.** 225 **C.** 80 **D.** 64 **E.** 104

(13) If $2\begin{bmatrix} 2 & 3 \\ -2 & 4 \end{bmatrix} + X = 3\begin{bmatrix} 4 & 3 \\ 1 & 5 \end{bmatrix}$, then $X =$
A. $\begin{bmatrix} 8 & 15 \\ 1 & 7 \end{bmatrix}$ **B.** $\begin{bmatrix} 8 & 3 \\ 7 & 7 \end{bmatrix}$ **C.** $\begin{bmatrix} 3 & 7 \\ 8 & 7 \end{bmatrix}$ **D.** $\begin{bmatrix} 7 & 7 \\ 8 & 3 \end{bmatrix}$ **E.** $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

(14) If $= \begin{bmatrix} cosx & sinx \\ -sinx & cosx \end{bmatrix}$, then $det(A^{-1}) =$
A. 0 **B.** 2 **C.** -2 **D.** 1 **E.** -1

(15) If *A* is a 2×2 matrix such that $det\ A = p^2$ and *k* is a real number, then $det(kA) =$
A. kp **B.** k^2p **C.** k^2p^2 **D.** k^4p **E.** k^4p^4

(16) A spinner is equally likely to point to any one of the numbers 1,2,3,4. If the spinner is spun two times, then the probability that two scores will be different is
A. $\frac{1}{2}$ **B.** $\frac{1}{3}$ **C.** $\frac{1}{4}$ **D.** $\frac{2}{3}$ **E.** $\frac{3}{4}$

(17) The probability of an event A is *3-6x*. Then *x* cannot be
A. $\frac{1}{2}$ **B.** $\frac{1}{3}$ **C.** $\frac{5}{12}$ **D.** $\frac{2}{5}$ **E.** $\frac{1}{5}$

(18) The probability that a bee will land on a target is $\frac{2}{5}$. It is found that when a bee lands on the target, the probability that it will attempt to sting is $\frac{1}{3}$. Then the probability that a bee will land on the target and attempt to sting is
A. $\frac{2}{15}$ **B.** $\frac{1}{5}$ **C.** $\frac{2}{5}$ **D.** $\frac{1}{8}$ **E.** $\frac{6}{5}$

(19) The opposite angles of a cyclic quadrilateral are in the ratio 3:7. The difference of their degree measure is
A. 72° **B.** 54° **C.** 36° **D.** 18° **E.** non of these

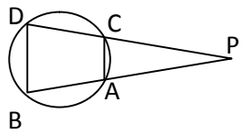
(20) PQ is a diameter of a circle. The chord LM cuts PQ at N. If LN=9cm, NM=4cm, NQ=2cm, then the diameter, in cm, of the circle is

A. 18 **B.** 10 **C.** 15 **D.** 20 **E.** 25

(21) The areas of two similar triangles are 56.25cm^2 and 42.25cm^2 respectively. Then the ratio of their altitudes is

A. 5:3 **B.** 13:14 **C.** 15:13 **D.** 16:14 **E.** non of these

(22) In the figure, AB=3, CD=6. If $\frac{\propto(\Delta PAC)}{\alpha(\Delta PDB)} = \frac{1}{4}$, then PC=



A. 2 **B.** 2.5 **C.** 3 **D.** 3.5 **E.** 4

(23) The position vectors of P and Q are $\begin{bmatrix}2\\3\end{bmatrix}$ and $\begin{bmatrix}5\\-1\end{bmatrix}$ respectively. The unit vector in the direction of $\overrightarrow{PQ}$ is

A. $-3\hat{\imath}+4\hat{\jmath}$ **B.** $-\frac{3}{5}\hat{\imath}+\frac{4}{5}\hat{\jmath}$ **C.** $\frac{3}{5}\hat{\imath}-\frac{4}{5}\hat{\jmath}$ **D.** $3\hat{\imath}-4\hat{\jmath}$ **E.** non of these

(24) Given that $\vec{a} = 3\hat{\imath}+4\hat{\jmath}$. Then the vector with magnitude 20 units and in the direction of $\vec{a}$ is

A. $12\hat{\imath}+16\hat{\jmath}$ **B.** $60\hat{\imath}+120\hat{\jmath}$ **C.** $-12\hat{\imath}-16\hat{\jmath}$ **D.** $9\hat{\imath}+12\hat{\jmath}$ **E.** $21\hat{\imath}+28\hat{\jmath}$

(25) In triangle ABC if α =30˚, γ=105˚ and b=8, then a=

A. $8\sqrt{3}$ **B.** $8\sqrt{2}$ **C.** $6\sqrt{2}$ **D.** $4\sqrt{3}$ **E.** $4\sqrt{2}$

(26) If $(sin\theta\text{-}cos\theta)^2=\frac{3}{4}$, then $sin2\theta$=

A. $\frac{2}{5}$ **B.** $\frac{\sqrt{3}}{2}$ **C.** 1 **D.** $\frac{1}{4}$ **E.** non of these

(27) Given that $p=cos\theta+sin\theta$, $q=cos\theta\text{-}sin\theta$, then $p^2\text{-}q^2$=

A. $2+2sin2\theta$ **B.** $sin2\theta$ **C.** $2\text{-}2sin2\theta$ **D.** $2sin2\theta$ **E.** $4sin2\theta$

(28) Given that $y = \frac{lnx^2}{3x}$, the value of $\frac{dy}{dx}$ when x=1 is

A. 1 **B.** $\frac{2}{3}$ **C.** $\frac{1}{3}$ **D.** -1 **E.** -2

(29) The maximum point of the curve $y=4x\text{-}x^2$ is

A. (2, -4) **B.** (-2, -4) **C.** (-2, 4) **D.** (2, 4) **E.** non of these

(30) $\lim_{x\to\infty}\frac{x^3(6-x)}{x^4+5} =$

A. 1 **B.** -1 **C.** $\frac{6}{5}$ **D.** $-\frac{1}{6}$ **E.** $-\frac{6}{5}$

2. Functions f and g are defined by $f(x)=4x\text{-}3$ and $g(x)=2x+1$. Find $(f.g)$ and $f^{-1}(x)$ in simplified forms. Show also that $(f.g)^{-1}(x) = g^{-1}(f^{-1}(x))$. **(10 Marks)**

3. Given that the expression $2x^3+px^2\text{-}8x+q$ is exactly divisible by $2x^2\text{-}7x+6$, evaluate p and q and factorize the expression completely. **(10 Marks)**

4. A geometric progression has three terms a, b, c whose sum is 42. If 6 is added to each of the first two terms and 3 to third, a new G.P results whose first term is the same as b. Find a, b and c. **(10 Marks)**

5. A die is rolled 360 times. Find the expected frequency of a factor of 6 and the expected frequency of a prime number. If all the scores obtained in these 360 trials are added together, what is the expected total score? **(10 Marks)**

6. ΔABC is inscribed in a circle. Straight lines are drawn through B and C parallel to CA and BA respectively, to meet the tangent at A in D and E. Prove that $\frac{DA}{AE} = \frac{AB}{EC} = \frac{AB^2}{AC^2}$ **(10 Marks)**

7. If α + β + θ = π, show that $\sin\alpha - \sin\beta + \sin\theta = 4\sin\frac{\alpha}{2}\cos\frac{\beta}{2}\sin\frac{\theta}{2}$ **(10 Marks)**

8. Determine the turning point on the curve $y=2x^3+3x^2\text{-}12x+7$ and state whether it is a maximum or a minimum. Then sketch the graph of the curve. **(10 Marks)**